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Nonlinear coupled fixed point theorems in ordered generalized metric spaces with integral type

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Abstract

In this article, we study coupled coincidence and coupled common fixed point theorems in ordered generalized metric spaces for nonlinear contraction condition related to a pair of altering distance functions. Our results generalize and modify several comparable results in the literature.

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Keywords: ordered set, coupled coincidence point, coupled common fixed point, generalized metric space, altering distance function, weakly contractive condition, contraction of integral type

1 Introduction

Fixed points of mappings in ordered metric space are of great use in many mathematical problems in applied and pure mathematics. The first result in this direction was obtained by Ran and Reurings [1], in this study the authors presented some applications of their obtained results to matrix equations. In [2,3], Nieto and López extended the result of Ran and Reurings [1] for non-decreasing mappings and applied their result to get a unique solution for a first order differential equation. While Agarwal et al. [4] and O'Regan and Petrutel [5] studied some results for a generalized contractions in ordered metric spaces. Bhaskar and Lakshmikantham [6] introduced the notion of a coupled fixed point of a mapping F from $X \times X$ into X. They established some coupled fixed point results and applied their results to the study of existence and uniqueness of solution for a periodic boundary value problem. Lakshmikantham and Ćirić [7] introduced the concept of coupled coincidence point and proved coupled coincidence and coupled common fixed point results for mappings F from $X \times X$ into X and g from X into X satisfying nonlinear contraction in ordered metric space. For the detailed survey on coupled fixed point results in ordered metric spaces, topological spaces, and fuzzy normed spaces, we refer the reader to [6-24].

On the other hand, in [25], Mustafa and Sims introduced a new structure of generalized metric spaces called G-metric spaces. In [26-32], some fixed point theorems for mappings satisfying different contractive conditions in such spaces were obtained. Abbas et al. [33] proved some coupled common fixed point results in two generalized metric spaces. While Shatanawi [34] established some coupled fixed point results in G-

metric spaces. Saadati et al. [35] established some fixed point in generalized ordered metric space. Recently, Choudhury and Maity [36] initiated the study of coupled fixed point in generalized ordered metric spaces.

In this article, we derive coupled coincidence and coupled common fixed point theorems in generalized ordered metric spaces for nonlinear contraction condition related to a pair of altering distance functions.

2 Basic concepts

Khan et al. [37] introduced the concept of altering distance function.

Definition 2.1. A function $\varphi : [0, +\infty) \to [0, +\infty)$ is called an *altering distance function* if the following properties are satisfied:

(1) φ is continuous and non-decreasing,
(2) $\varphi(t) = 0$ if and only if $t = 0$.

For more details on the following definitions and results, we refer the reader to Mustafa and Sims [25].

Definition 2.2. Let X be a non-empty set and let $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying the following properties:

(G1) $G(x, y, z) = 0$ if and only if $x = y = z$,
(G2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,
(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,
(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \ldots$(: symmetry in all three variables),
(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

Then the function G is called a *generalized metric* or, more specifically, a *G-metric* on X and the pair (X, G) is called a *G-metric space*.

Definition 2.3. Let (X, G) be a G-metric space and (x_n) be a sequence in X. We say that (x_n) is *G-convergent* to a point $x \in X$ or (x_n) G-converges to x if, for any $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x, x_n, x_m) < \varepsilon$ for all $m, n \geq k$, that is, $\lim_{n,m\to+\infty} G(x, x_n, x_m) = 0$. In this case, we write $x_n \to x$ or $\lim_{n\to+\infty} x_n = x$.

Proposition 2.1. *Let (X, G) be a G-metric space. Then the following are equivalent:*

(1) *(x_n) is G-convergent to x.*
(2) *$G(x_n, x_n, x) \to 0$ as $n \to +\infty$.*
(3) *$G(x_n, x, x) \to 0$ as $n \to +\infty$.*
(4) *$G(x_n, x_m, x) \to 0$ as $n, m \to +\infty$.*

Definition 2.4. Let (X, G) be a G-metric space and (x_n) be a sequence in X. We say that (x_n) is a *G-Cauchy sequence* if, for any $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \varepsilon$ for all $n, m, l \geq k$, that is, $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to +\infty$.

Proposition 2.2. *Let (X, G) be a G-metric space. Then the following are equivalent:*

(1) *The sequence (x_n) is a G-Cauchy sequence.*
(2) *For any $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$ for all $n, m \geq k$.*

Definition 2.5. Let (X, G) and (X', G') be two G-metric spaces. We say that a function $f : (X, G) \to (X', G')$ is *G-continuous* at a point $a \in X$ if and only if, for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$x, y \in X, \quad G(a, x, y) < \delta \Rightarrow G'(f(a), f(x), f(y)) < \varepsilon.$$

A function f is *G-continuous* on X if and only if it is G-continuous at every point $a \in X$.

Proposition 2.3. *Let* (X, G) *be a G-metric space. Then the function G is jointly continuous in all three of its variables.*

We give some examples of G-metric spaces.

Example 2.1. Let $(\mathbb{R}, d)$ be the usual metric space. Define a function $G_s:\mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by

$$G_s(x, y, z) = d(x, y) + d(y, z) + d(x, z)$$

for all $x, y, z \in \mathbb{R}$. Then it is clear that $(\mathbb{R}, G_s)$ is a G-metric space.

Example 2.2. Let $X = \{a, b\}$. Define a function G: $X \times X \times X$ $\mathbb{R}$ *by*

$$G(a, a, a) = G(b, b, b) = 0, \quad G(a, a, b) = 1, \quad G(a, b, b) = 2$$

and extend G *to* $X \times X \times X$ *by* using the symmetry in the variables. Then it is clear that (X, G) is a G-metric space.

Definition 2.6. A G-metric space (X, G) is said to be *G-complete* if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).

For more details about the following definitions, we refer the reader to [6,7].

Definition 2.7. Let X be a non-empty set and $F : X \times X \to X$ be a given mapping. An element $(x, y) \in X \times X$ is called a *coupled fixed point* of F if $F(x, y) = x$ and $F(y, x) = y$.

Definition 2.8. Let $(X, \leq)$ be a partially ordered set. A mapping $F : X \times X \to X$ *is* said to have the *mixed monotone property* if $F(x,y)$ is monotone non-decreasing in x and is monotone non-increasing in y, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, x_1 \leq x_2 \Rightarrow F(x_1, y) \leq F(x_2, y)$$

and

$$y_1, y_2 \in X, y_1 \leq y_2 \Rightarrow F(x, y_2) \leq F(x, y_1).$$

Lakshmikantham and Ćirić [7] introduced the concept of a g-mixed monotone mapping.

Definition 2.9. Let $(X, \leq)$ be a partially ordered set, $F : X \times X \to X$ *and* $g : X \to X$ *be* mappings. The mapping F is said to have the *mixed g-monotone property* if $F(x, y)$ is monotone g-non-decreasing in x and is monotone g-non-increasing in y, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, gx_1 \leq gx_2 \Rightarrow F(x_1, y) \leq F(x_2, y)$$

and

$$y_1, y_2 \in X, gy_1 \leq gy_2 \Rightarrow F(x, y_2) \leq F(x, y_1).$$

Definition 2.10. Let X be a non-empty set, $F : X \times X \to X$ and $g : X \to X$ be mappings. An element $(x, y) \in X \times X$ is called a *coupled coincidence point* of F and g if $F(x, y) = gx$ and $F(y, x) = gy$.

Definition 2.11. Let X be a non-empty set, $F : X \times X \to X$ *and* $g : X \to X$ *be* mappings. An element $(x, y) \in X \times X$ is called a *coupled common fixed point* of F and g if $F(x, y) = gx = x$ and $F(y, x) = gy = y$.

Definition 2.12. Let X be a non-empty set, $F : X \times X \to X$ *and* g*:* $X \to X$ *be* mappings. We say that F and g are *commutative* if $g(F(x, y)) = F(gx, gy)$ for all $x, y \in X$.

Definition 2.13. Let X be a non-empty set, $F : X \times X \to X$ and $g : X \to X$ be mappings. Then F and g are said to be *weak* compatible* (or *w*-compatible*) *if* $g(F(x, x)) = F(gx, gx)$ whenever $g(x) = F(x, x)$.

3 Main results

The following is the first result.

Theorem 3.1. *Let* $(X, \leq)$ *be a partially ordered set and* (X, G) *be a complete G-metric space. Let* $F : X \times X \to X$ *and* $g : X \to X$ *be continuous mappings such that F has the mixed g-monotone property and g commutes with F. Assume that there are altering distance functions* ψ *and* φ *such that*

$$\begin{aligned}&\psi(G(F(x,y),F(u,v),F(w,z)))\\ &\leq \psi(\max\{G(gx,gu,gw),G(gy,gv,gz)\})-\phi(\max\{G(gx,gu,gw),G(gy,gv,gz)\})\end{aligned} \tag{1}$$

for all $x, y, u, v, w, z \in X$ *with* $gw \leq gu \leq gx$ *and* $gy \leq gv \leq gz$*. Also, suppose that* $F(X \times X) \subseteq g(X)$*. If there exist* $x_0, y_0 \in X$ *such that* $gx_0 \leq F(x_0, y_0)$ *and* $F(y_0, x_0) \leq gy_0$*, then F and g have a coupled coincidence point.*

Proof. Let $x_0, y_0 \in X$ such that $gx_0 \leq F(x_0, y_0)$ and $F(y_0, x_0) \leq gy_0$. Since we have $F(X \times X) \subseteq g(X)$, we can choose $x_1, y_1 \in X$ such that $gx_1 = F(x_0, y_0)$ and $gy_1 = F(y_0, x_0)$. Again, since $F(X \times X) \subseteq g(X)$, we can choose $x_2, y_2 \in X$ such that $gx_2 = F(x_1, y_1)$ and $gy_2 = F(y_1, x_1)$. Since F has the mixed g-monotone property, we have $gx_0 \leq gx_1 \leq gx_2$ and $gy_2 \leq gy_1 \leq gy_0$. Continuing this process, we can construct two sequences (x_n) and (y_n) in X such that

$$gx_n = G(x_{n-1}, y_{n-1}) \leq gx_{n+1} = F(x_n, y_n)$$

and

$$gy_{n+1} = F(y_n, x_n) \leq gy_n = F(y_{n-1}, x_{n-1}).$$

If, for some integer n, we have $(gx_{n+1}, gy_{n+1}) = (gx_n, gy_n)$, then $F(x_n, y_n) = gx_n$ and $F(y_n, x_n) = gy_n$, that is, (x_n, y_n) is a coincidence point of F and g. So, from now on, we assume that $(gx_{n+1}, gy_{n+1}) \neq (gx_n, gy_n)$ for all $n \in \mathbb{N}$, that is, we assume that either $gx_{n+1} \neq gx_n$ or $gy_{n+1} \neq gy_n$.

We complete the proof with the following steps.

Step 1: We show that

$$\lim_{n\to+\infty} \max\{G(gx_{n-1}, gx_n, gx_n\}, G(gy_{n-1}, gy_n, gy_n)\} = 0. \tag{2}$$

For each $n \in \mathbb{N}$, using the inequality (1), we obtain

$$\begin{aligned}\psi(G(gx_{n+1}, gx_{n+1}, gx_n)) &= \psi(G(F(x_n, y_n), F(x_n, y_n), F(x_{n-1}, y_{n-1}))) \\ &\le \psi(\max\{G(gx_n, gx_n, gx_{n-1}), G(gy_n, gy_n, gy_{n-1}))\}) \\ &\quad - \phi(\max\{G(gx_n, gx_n, gx_{n-1}), G(gy_n, gy_n, gy_{n-1}))\}) \\ &\le \psi(\max\{G(gx_n, gx_n, gx_{n-1}), G(gy_n, gy_n, gy_{n-1}))\}).\end{aligned} \tag{3}$$

Since ψ is a non-decreasing function, we get

$$G(gx_{n+1}, gx_{n+1}, gx_n) \le \max\{G(gx_n, gx_n, gx_{n-1}), G(gy_n, gy_n, gy_{n-1}))\}. \tag{4}$$

On the other hand, we have

$$\begin{aligned}\psi(G(gy_n, gy_{n+1}, gy_{n+1})) &= \psi(G(F(y_{n-1}, x_{n-1}), F(y_n.x_n), F(y_n, x_n))) \\ &\le \psi(\max\{G(gx_{n-1}, gx_n, gx_n), G(gy_{n-1}, gy_n, gy_n)\}) \\ &\quad - \phi(\max\{G(gx_{n-1}, gx_n, gx_n), G(gy_{n-1}, gy_n, gy_n)\}) \\ &\le \psi(\max\{G(gx_{n-1}, gx_n, gx_n), G(gy_{n-1}, gy_n, gy_n)\}).\end{aligned} \tag{5}$$

Since ψ is a non-decreasing function, we get

$$G(gy_n, gy_{n+1}, gy_{n+1}) \le \max\{G(gx_{n-1}, gx_n, gx_n), G(gy_{n-1}, gy_n, gy_n)\}. \tag{6}$$

Thus, by (4) and (6), we have

$$\begin{aligned}&\max\{G(gx_n, gx_{n+1}, gx_{n+1}), G(gy_n, gy_{n+1}, gy_{n+1})\} \\ &\le \max\{G(gx_{n-1}, gx_n, gx_n), G(gy_{n-1}, gy_n, gy_n)\}.\end{aligned}$$

Thus $(\max\{G(gx_{n-1}, gx_n, gx_n), G(gy_{n-1}, gy_n, gy_n)\})$ is a non-negative decreasing sequence. Hence, there exists $r \ge 0$ such that

$$\lim_{n \to +\infty} \max\{G(gx_{n-1}, gx_n, gx_n), G(gy_{n-1}, gy_n, gy_n)\} = r.$$

Now, we show that $r = 0$. Since $\varphi : [0, +\infty) \to [0, +\infty)$ is a non-decreasing function, then, for any $a, b \in [0, +\infty)$, we have $\psi(\max\{a, b\}) = \max\{\psi(a), \psi(b)\}$. Thus, by (3)) and (5), we have

$$\begin{aligned}&\psi(\max\{G(gx_{n-1}, gx_n, gx_n), G(gy_{n-1}, gy_n, gy_n)\}) \\ &= \max\{\psi(G(gx_{n-1}, gx_n, gx_n)), \psi(G(gy_{n-1}, gy_n, gy_n))\} \\ &\le \psi(\max\{G(gx_{n-1}, gx_n, gx_n), G(gy_{n-1}, gy_n, gy_n)\}) \\ &\quad -\phi(\max\{G(gx_{n-1}, gx_n, gx_n), G(gy_{n-1}, gy_n, gy_n)\}).\end{aligned}$$

Letting $n \to +\infty$ in the above inequality and using the continuity of ψ, we get

$$\psi(r) \le \psi(r) - \phi(r).$$

Hence $\varphi(r) = 0$. Thus $r = 0$ and (2) holds.

Step 2: We show that (gx_n) and (gy_n) are G-Cauchy sequences. Assume that (x_n) or (y_n) is not a G-Cauchy sequence, that is,

$$\lim_{n,m \to +\infty} G(gx_m, gx_n, gx_n) \neq 0$$

or

$$\lim_{n,m\to+\infty} G(gy_m, gy_n, gy_n) \neq 0.$$

This means that there exists $\epsilon > 0$ for which we can find subsequences of integers $(m(k))$ and $(n(k))$ with $n(k) > m(k) > k$ such that

$$\max\{G((gx_{m(k)}), G(gx_{n(k)}), G(gx_{n(k)})), G((gy_{m(k)}), G(gy_{n(k)}), G(gy_{n(k)}))\} \geq \varepsilon. \tag{7}$$

Further, corresponding to $m(k)$ we can choose $n(k)$ in such a way that it is the smallest integer with $n(k) > m(k)$ and satisfying (7). Then we have

$$\max\{G((gx_{m(k)}), G(gx_{n(k)-1}), G(gx_{n(k)-1})), G((gy_{m(k)}), G(gy_{n(k)-1}), G(gy_{n(k)-1}))\} < \varepsilon. \tag{8}$$

Thus, by (G_5) and (8), we have

$$\begin{aligned}
& G(gx_{m(k)}, gx_{n(k)}, gx_{n(k)}) \\
\leq\ & G(gx_{m(k)}, gx_{n(k)-1}, gx_{n(k)-1}) + G(gx_{n(k)-1}, gx_{n(k)}, gx_{n(k)}) \\
\leq\ & G(gx_{m(k)}, gx_{m(k)-1}, gx_{m(k)-1}) + G(gx_{m(k)-1}, gx_{n(k)-1}, gx_{n(k)-1}) \\
& + G(gx_{n(k)-1}, gx_{n(k)}, gx_{n(k)}) \\
\leq\ & 2G(gx_{m(k)}, gx_{m(k)}, gx_{m(k)-1}) + G(gx_{m(k)-1}, gx_{n(k)-1}, gx_{n(k)-1}) \\
& + G(gx_{n(k)-1}, gx_{n(k)}, gx_{n(k)}) \\
<\ & 2G(gx_{m(k)}, gx_{m(k)}, gx_{m(k)-1}) + \varepsilon + G(gx_{n(k)-1}, gx_{n(k)}, gx_{n(k)}).
\end{aligned}$$

Thus, by (2), we have

$$\limsup_{k\to+\infty} G(gx_{m(k)}, gx_{n(k)}, gx_{n(k)}) \leq \limsup_{k\to+\infty} G(gx_{m(k)-1}, gx_{n(k)-1}, gx_{n(k)-1}) \leq \varepsilon. \tag{9}$$

Similarly, we have

$$\limsup_{k\to+\infty} G(gy_{m(k)}, gy_{n(k)}, gy_{n(k)}) \leq \limsup_{k\to+\infty} G(gy_{m(k)-1}, gy_{n(k)-1}, gy_{n(k)-1}) \leq \varepsilon. \tag{10}$$

Thus, by (9) and (10), we have

$$\begin{aligned}
& \limsup_{k\to+\infty} \max\{G(gx_{m(k)}, gx_{n(k)}, gx_{n(k)}), G(gy_{m(k)}, gy_{n(k)}, gy_{n(k)})\} \\
\leq\ & \limsup_{k\to+\infty} \max\{G(gx_{m(k)-1}, gx_{n(k)-1}, gx_{n(k)-1}), G(gy_{m(k)-1}, gy_{n(k)-1}, gy_{n(k)-1})\} \\
\leq\ & \varepsilon.
\end{aligned}$$

Using (7), we get

$$\begin{aligned}
& \limsup_{k\to+\infty} \max\{G(gx_{m(k)}, gx_{n(k)}, gx_{n(k)}), G(gy_{m(k)}, gy_{n(k)}, gy_{n(k)})\} \\
=\ & \limsup_{k\to+\infty} \max\{G(gx_{m(k)-1}, gx_{n(k)-1}, gx_{n(k)-1}), G(gy_{m(k)-1}, gy_{n(k)-1}, gy_{n(k)-1})\} \\
=\ & \varepsilon.
\end{aligned} \tag{11}$$

Now, using the inequality (1), we obtain

$$\begin{aligned}
& \psi(G(gx_{n(k)}, gx_{n(k)}, G(gx_{m(k)})) \\
=\ & \psi(G(F(x_{n(k)-1}, y_{n(k)-1}), F(x_{n(k)-1}, y_{n(k)-1}), F(x_{m(k)-1}, y_{m(k)-1}))) \\
\leq\ & \psi(\max\{G(gx_{n(k)-1}, gx_{n(k)-1}, gx_{m(k)-1}), G(gy_{n(k)-1}, y_{n(k)-1}, y_{m(k)-1})\}) \\
& - \phi(\max\{G(gx_{n(k)-1}, gx_{n(k)-1}, gx_{m(k)-1}), G(gy_{n(k)-1}, y_{n(k)-1}, y_{m(k)-1})\})
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
&\psi(G(gy_{m(k)}, gy_{n(k)}, gy_{n(k)})) \\
&= \psi(G(F(y_{m(k)-1}, x_{m(k)-1}), F(y_{n(k)-1}, x_{n(k)-1}), F(y_{n(k)-1}, x_{n(k)-1}))) \\
&\leq \psi(\max\{G\{gy_{m(k)-1}, gy_{n(k)-1}, gy_{n(k)-1}), G(gx_{m(k)_{-1}}, gx_{n(k)-1}, gx_{n(k)-1})\}) \\
&\quad - \phi(\max\{G(gy_{m(k)-1}, gy_{n(k)-1}, gy_{n(k)-1}), G(gx_{m(k)-1}, gx_{n(k)-1}, gx_{n(k)-1})\}).
\end{aligned} \tag{13}$$

Thus, by (12) and (13), we get

$$\begin{aligned}
&\psi(\max\{G(gx_{m(k)}, gx_{n(k)}, gx_{n(k)}), G(gy_{m(k)}, gy_{n(k)}, gy_{n(k)})\}) \\
&= \max\{\psi(G(gx_{m(k)}, gx_{n(k)}, gx_{n(k)})), \psi(G(gy_{m(k)}, gy_{n(k)}, gy_{n(k)}))\} \\
&\leq \psi(\max\{G(gy_{m(k)-1}, gy_{n(k)-1}, gy_{n(k)-1}), G(gx_{m(k)-1}, gx_{n(k)-1}, gx_{n(k)-1})\}) \\
&\quad - \phi(\max\{G(gy_{m(k)-1}, gy_{n(k)-1}, gy_{n(k)-1}), G(gx_{m(k)-1}, gx_{n(k)-1}, gx_{n(k)-1})\}).
\end{aligned}$$

Letting $k \to +\infty$ in the above inequality and using (11) and the fact that ψ and φ are continuous, we get

$$\psi(\varepsilon) \leq \psi(\varepsilon) - \phi(\varepsilon).$$

Hence $\varphi(\epsilon) = 0$ and so $\epsilon = 0$, which is a contradiction. Therefore, (gx_n) and (gy_n) are G-Cauchy sequences.

Step 3: The existence of a coupled coincidence point. Since (gx_n) and (gy_n) are G-Cauchy sequences in a complete G-metric space (X, G), there exist $x, y \in X$ such that (gx_n) and (gyn) are G-convergent to points x and y, respectively, that is,

$$\lim_{n\to+\infty} G(gx_n, gx_n, x) = \lim_{n\to+\infty} G(gx_n, x, x) = 0 \tag{14}$$

and

$$\lim_{n\to+\infty} G(gy_n, gy_n, y) = \lim_{n\to+\infty} G(gy_n, y, y) = 0. \tag{15}$$

Then, by (14), (15) and the continuity of g, we have

$$\lim_{n\to+\infty} G(g(gx_n), g(gx_n), gx) = \lim_{n\to+\infty} G(g(gx_n), gx, gx) = 0 \tag{16}$$

and

$$\lim_{n\to+\infty} G(g(gy_n), g(gy_n), gy) = \lim_{n\to+\infty} G(g(gy_n), gy, gy) = 0. \tag{17}$$

Therefore, $(g(gx_n))$ is G-convergent to gx and $(g(gy_n))$ is G-convergent to gy. Since F and g commute, we get

$$g(gx_{n+1}) = g(F(x_n, y_n)) = F(gx_n, gy_n) \tag{18}$$

and

$$g(gy_{n+1}) = g(F(y_n, x_n)) = F(gy_n, gx_n). \tag{19}$$

Using the continuity of F and letting $n \to +\infty$ in (18) and (19), we get $gx = F(x, y)$ and $gy = F(y, x)$. This implies that (x, y) is a coupled coincidence point of F and g. This completes the proof.

Tacking $g = I_X$ (: the identity mapping) in Theorem 3.1., we obtain the following coupled fixed point result.

Corollary 3.1. *Let $(X, \leq)$ be a partially ordered set and (X, G) be a complete G-metric space. Let $F : X \times X \to X$ be a continuous mapping satisfying the mixed monotone property. Assume that there exist the altering distance functions ψ and φ such that*

$$\psi(G(F(x,y),F(u,v),F(w,z))) \\ \leq \psi(\max\{G(x,u,w),G(y,v,z)\}) - \phi(\max\{G(x,u,w),G(y,v,z)\})$$

for all $x, y, u, v, w, z \in X$ with $w \leq u \leq x$ and $y \leq v \leq z$. If there exist $x_0, y_0 \in X$ such that $x_0 \leq F(x_0, y_0)$ and $F(y_0, x_0) \leq y_0$, then F has a coupled fixed point.

Now, we derive coupled coincidence point results without the continuity hypothesis of the mappings *F, g* and the commutativity hypothesis of *F, g*. However, we consider the additional assumption on the partially ordered set $(X, \leq)$.

We need the following definition.

Definition 3.1. Let $(X, \leq)$ be a partially ordered set and G be a G-metric on X. We say that $(X, G, \leq)$ is *regular* if the following conditions hold:

(1) if a non-decreasing sequence (x_n) is such that $x_n \to x$, then $x_n \leq x$ for all $n \in \mathbb{N}$,
(2) if a non-increasing sequence (y_n) is such that $y_n \to y$, then $y \leq y_n$ for all $n \in \mathbb{N}$.

The following is the second result.

Theorem 3.2. *Let $(X, \leq)$ be a partially ordered set and G be a G-metric on $\times$ such that $(X, G, \leq)$ is regular. Assume that there exist the altering distance functions ψ, φ and mappings $F : X \times X \to X$ and $g: X \to X$ such that*

$$\psi(G(F(x,y),F(u,v),F(w,z))) \\ \leq \psi(\max\{G(gx,gu,gw),G(gy,gv,gz)\}) - \phi(\max\{G(gx,gu,gw),G(gy,gv,gz)\})$$

for all $x, y, u, v, w, z \in X$ with $gw \leq gu \leq gx$ and $gy \leq gv \leq gz$. Suppose also that $(g(X), G)$ is G-complete, F has the mixed g-monotone property and $F(X \times X) \subseteq g(X)$. If there exist $x_0, y_0 \in X$ such that $gx_0 \leq F(x_0, y_0)$ and $F(y_0, x_0) \leq gy_0$, then F and g have a coupled coincidence point.

Proof. Following Steps 1 and 2 in the proof of Theorem 3.1., we know that (gx_n) and (gy_n) are G-Cauchy sequences in $g(X)$ with $gx_n \leq gx_{n+1}$ and $gy_n \geq gy_{n+1}$ for all $n \in \mathbb{N}$. Since $(g(X), G)$ is G-complete, there exist $x, y \in X$ such that $gx_n \to gx$ and $gy_n \to gy$. Since $(X, G, \leq)$ is regular, we have $gx_n \leq gx$ and $gy \leq gy_n$ for all $n \in \mathbb{N}$. Thus we have

$$\begin{aligned}\psi(G(F(x,y),gx_{n+2},gx_{n+1})) &= \psi(G(F(x,y),F(x_{n+1},y_{n+1}),F(gx_n,gy_n))) \\ &\leq \psi(\max\{G(gx,gx_{n+1},gx_n),G(gy,gy_{n+1},gx_n)\}) \\ &\quad - \phi(\max\{G(gx,gx_{n+1},gx_n),G(gy,gy_{n+1},gy_n)\}).\end{aligned}$$

Letting $n \to +\infty$ in the above inequality and using the continuity of ψ and φ, we obtain $\psi(G(F(x, y),gx, gx)) = 0$, which implies that $G(F(x, y), gx, gx) = 0$. Therefore, $F(x, y) = gx$.

Similarly, one can show that $F(y, x) = gy$. Thus (x, y) is a coupled coincidence point of F and g, this completes the proof.

Tacking $g = I_X$ in Theorem 3.2., we obtain the following result.

Corollary 3.2. *Let $(X, \leq)$ be a partially ordered set and G be a G-metric on X such that $(X, G, \leq)$ is regular and (X, G) is G-complete. Assume that there exist the altering distance functions ψ, φ and a mapping*

$F : X \times X \to X$ having the mixed monotone property such that

$$\psi(G(F(x,y), F(u,v), F(w,z))) \\ \leq \psi(\max\{G(x,u,w), G(y,v,z)\}) - \phi(\max\{G(x,u,w), G(y,v,z)\})$$

for all $x, y, u, v, w, z \in X$ with $w \leq u \leq x$ and $y \leq v \leq z$. If there exist $x_0, y_0 \in X$ such that $x_0 \leq F(x_0, y_0)$ and $F(y_0, x_0) \leq y_0$, then F has a coupled fixed point.

Now, we prove the existence and uniqueness theorem of a coupled common fixed point. If $(X, \leq)$ is a partially ordered set, we endow the product set $X \times X$ with the partial order defined by

$$(x,y) \leq (u,v) \quad \Leftrightarrow \quad x \leq u, \ v \leq y.$$

Theorem 3.3. *In addition to the hypotheses of Theorem* 3.1., *suppose that, for any $(x, y), (x^*, y^*) \in X \times X$, there exists $(u, v) \in X \times X$ such that $(F(u, v), F(v, u))$ is comparable with $(F(x, y), F(y, x))$ and $(F(x^*, y^*), F(y^*, x^*))$. Then F and g have a unique coupled common fixed point, that is, there exists a unique $(x, y) \in X \times X$ such that $x = gx = F(x, y)$ and $y = gy = F(y, x)$.*

Proof. From Theorem 3.1., the set of coupled coincidence points is non-empty. We shall show that if (x, y) and (x^*, y^*) are coupled coincidence points, then

$$gx = gx^*, \quad gy = gy^*. \tag{20}$$

By the assumption, there exists $(u, v) \in X \times X$ such that $(F(u, v), F(v, u))$ is comparable to $(F(x, y), F(y, x))$ and $(F(x^*, y^*), F(y^*, x^*))$. Without the restriction to the generality, we can assume that $(F(x, y), F(y, x)) \leq (F(u, v), F(v, u))$ and $(F(x^*, y^*), F(y^*, x^*)) \leq (F(u, v), F(v, u))$. Put $u_0 = u$, $v_0 = v$ and choose $u_1, v_1 \in X$ so that $gu_1 = F(u_0, v_0)$ and $gv_1 = F(v_0, u_0)$. As in the proof of Theorem 3.1., we can inductively define the sequences (u_n) and (v_n) such that

$$gu_{n+1} = F(u_n, v_n), \quad gv_{n+1} = F(v_n, u_n).$$

Further, set $x_0 = x, y_0 = y, \ x_0^* = x^*, \ y_0^* = y^*$ and, by the same way, define the sequences (x_n), (y_n) and $(x_n^*), (y_n^*)$. Since $(gx, gy) = (F(x, y), F(y, x)) = (gx_1, gy_1)$ and $(F(u, v), F(v, u)) = (gu_1, gv_1)$ are comparable, $gx \leq gu_1$ and $gv_1 \leq gy$. One can show, by induction, that

$$gx \leq gu_n, \quad gv_n \leq gy$$

for all $n \in \mathbb{N}$. From (1), we have

$$\begin{aligned}\psi(G(gx, gx, gu_{n+1})) &= \psi(G(F(x,y), F(x,y), F(u_n, v_n))) \\ &\leq \psi(\max\{G(gx, gx, gu_n), G(gy, gy, gv_n)\}) \\ &\quad - \phi(\max\{G(gx, gx, gu_n), G(gy, gy, gv_n)\})\end{aligned}$$

and

$$\begin{aligned}\psi(G(gy, gy, gv_{n+1})) &= \psi(G(F(y,x), F(y,x), F(v_n, u_n))) \\ &\leq \psi(\max\{G(gx, gx, gu_n), G(gy, gy, gv_n)\}) \\ &\quad - \phi(\max\{G(gx, gx, gu_n), G(gy, gy, gv_n)\}).\end{aligned}$$

Hence it follows that

$$\begin{aligned}
&\psi(\max\{G(gx, gx, gu_{n+1}), G(gy, gy, gv_{n+1})\}) \\
&= \max\{\psi(G(gx, gx, gu_{n+1})), \psi(G(gy, gy, gv_{n+1}))\} \\
&\leq \psi(\max\{G(gx, gx, gu_n), G(gy, gy, gv_n)\}) \\
&\quad - \phi(\max\{G(gx, gx, gu_n), G(gy, gy, gv_n)\}) \\
&\leq \psi(\max\{G(gx, gx, gu_n), G(gy, gy, gv_n)\}).
\end{aligned}$$

Since ψ is non-decreasing, it follows that $(max\{G(gx, gx, gu_n), G(gy, gy, gv_n)\})$ is a decreasing sequence.

Hence there exists a non-negative real number r such that

$$\lim_{n\to+\infty} \max\{G(gx, gx, gu_n), G(gy, gy, gv_n)\} = r. \tag{21}$$

Using (21) and letting $n \to +\infty$ in the above inequality, we get

$$\psi(r) \leq \psi(r) - \phi(r).$$

Therefore, $\varphi(r) = 0$ and hence $r = 0$. Thus

$$\lim_{n\to+\infty} G(gx, gx, gu_n) = \lim_{n\to+\infty} G(gy, gy, gv_n)\} = 0. \tag{22}$$

Similarly, we can show that

$$\lim_{n\to+\infty} G(gx^*, gx^*, gu_{n+1}) = \lim_{n\to+\infty} G(gy^*, gy^*, gv_{n+1}) = 0. \tag{23}$$

Thus, by (G_5), (22), and (23), we have, as $n \to +\infty$,

$$G(gx, gx, gx^*) \leq G(gx, gx, gu_{n+1}) + p(gu_{n+1}, gu_{n+1}, gx^*) \to 0$$

and

$$G(gy, gy, gy^*) \leq G(gy, gy, gv_{n+1}) + G(gv_{n+1}, gv_{n+1}, gy^*) \to 0.$$

Hence $gx = gx^*$ and $gy = gy^*$. Thus we proved (20).

On the other hand, since $gx = F(x, y)$ and $gy = F(y, x)$, by commutativity of F and g, we have

$$g(gx) = g(F(x, y)) = F(gx, gy), \quad g(gy) = g(F(y, x)) = F(gy, gx). \tag{24}$$

Denote $gx = z$ and $gy = w$. Then, from (24), it follows that

$$gz = F(z, w), \quad gw = F(w, z). \tag{25}$$

Thus (z, w) is a coupled coincidence point. Then, from (20) with $x^* = z$ and $y^* = w$, it follows that $gz = gx$ and $gw = gy$, that is,

$$gz = z, \quad gw = w. \tag{26}$$

Thus, from (25) and (26), we have $z = gz = F(z, w)$ and $w = gw = F(w, z)$. Therefore, (z, w) is a coupled common fixed point of F and g.

To prove the uniqueness of the point (z, w), assume that (s, t) is another coupled common fixed point of F and g. Then we have

$$s = gs = F(s, t), \quad t = gt = F(t, s).$$

Since the pair (s, t) is a coupled coincidence point of F and g, we have $gs = gx = z$ and $gt = gy = w$. Thus $s = gs = gz = z$ and $t = gt = gw = w$. Hence, the coupled fixed point is unique. this completes the proof.

Now, we present coupled coincidence and coupled common fixed point results for mappings satisfying contractions of integral type. Denote by Λ the set of functions $\alpha : [0, +\infty) \to [0, +\infty)$ satisfying the following hypotheses:

(h1) α is a Lebesgue integrable mapping on each compact subset of $[0, +\infty)$,

(h2) for any $\varepsilon > 0$, we have $\int_0^{\varepsilon} \alpha(s)\mathrm{d}s > 0$.

Finally, we give the following results.

Theorem 3.4. *Let $(X, \leq)$ be a partially ordered set and (X, G) be a complete G-metric space. Let $F : X \times X \to X$ and $g : X \to X$ be continuous mappings such that F has the mixed g-monotone property and g commutes with F. Assume that there exist $\alpha, \beta \in \Lambda$ such that*

$$\int_0^{G(F(x,y),F(u,v),F(w,z))} \alpha(s)\mathrm{d}s \leq \int_0^{\max\{G(gx,gu,gw),G(gy,gv,gz)\}} \alpha(s)\mathrm{d}s - \int_0^{\max\{G(gx,gu,gw),G(gy,gv,gz)\}} \beta(s)\mathrm{d}s$$

for all $x, y, u, v, w, z \in X$ with $gw \leq gu \leq gx$ and $gy \leq gv \leq gz$. Also, suppose that $F(X \times X) \subseteq g(X)$. If there exist $x_0, y_0 \in X$ such that $gx_0 \leq F(x_0, y_0)$ and $F(y_0, x_0) \leq gy_0$, then F and g have a coupled coincidence point.

Proof. We consider the functions $\psi, \varphi : [0, +\infty) \to [0, +\infty)$ defined by

$$\psi(t) = \int_0^t \alpha(s)\mathrm{d}s, \quad \phi(t) = \int_0^t \beta(s)\mathrm{d}s$$

for all $t \geq 0$. It is clear that ψ and φ are altering distance functions. Then the results follow immediately from Theorem 3.1.. This completes the proof.

Corollary 3.3. *Let $(X, \leq)$ be a partially ordered set and (X, G) be a complete G-metric space. Let $F : X \times X \to X$ be a continuous mappings satisfying the mixed monotone property. Assume that there exist $\alpha, \beta \in \Lambda$ such that*

$$\int_0^{G(F(x,y),F(u,v),F(w,z))} \alpha(s)\mathrm{d}s \leq \int_0^{\max\{G(x,u,w),G(y,v,z)\}} \alpha(s)\mathrm{d}s - \int_0^{\max\{G(x,u,w),G(y,v,z)\}} \beta(s)\mathrm{d}s$$

for all $x, y, u, v, w, z \in X$ with $w \leq u \leq x$ and $y \leq v \leq z$. If there exist $x_0, y_0 \in X$ such that $x_0 \leq F(x_0, y_0)$ and $F(y_0, x_0) \leq y_0$, then F has a coupled fixed point.

Proof. Tacking $g = I_X$ in Theorem 3.3., we obtain Corollary 3.3..

Putting $\beta(s) = (1 - k)\alpha(s)$ with $k \in [0,1)$ in Theorem 3.3., we obtain the following result.

Corollary 3.4. *Let $(X, \leq)$ be a partially ordered set and (X, G) be a complete G-metric space. Let $F : X \times X \to X$ and g: $X \to X$ be continuous mappings such that F has the mixed g-monotone property and g commutes with F. Assume that there exist $\alpha \in \Lambda$ and $k \in [0, 1)$ such that*

$$\int_0^{G(F(x,y),F(u,v),F(w,z))} \alpha(s)\mathrm{d}s \leq k \int_0^{\max\{G(gx,gu,gw),G(gy,gv,gz)\}} \alpha(s)\mathrm{d}s$$

for all $x, y, u, v, w, z \in X$ with $gw \leq gu \leq gx$ and $gy \leq gv \leq gz$. Also, suppose that $F(X \times X) \subseteq g(X)$.

If there exist $x_0, y_0 \in X$ such that $gx_0 \leq F(x_0, y_0)$ and $F(y_0, x_0) \leq gy_0$, then F and g have a coupled coincidence point.

Tacking $\alpha(s) = 1$ in Corollary 3.4., we obtain the following result.

Corollary 3.5. *Let $(X, \leq)$ be a partially ordered set and (X, G) be a complete G-metric space. Let $F : X \times X \to X$ and $g : X \to X$ be continuous mappings such that F has the mixed g-monotone property and g commutes with F. Assume that there exists $k \in [0, 1)$ such that*

$$G(F(x,y), F(u,v), F(w,z)) \leq k \max\{G(gx, gu, gw), G(gy, gv, gz)\}$$

for all $x, y, u, v, w, z \in X$ with $gw \leq gu \leq gx$ and $gy \leq gv \leq gz$. Also, suppose that $F(X \times X) \subseteq g(X)$. If there exist $x_0, y_0 \in X$ such that $gx_0 \leq F(x_0, y_0)$ and $F(y_0, x_0) \leq gy_0$, then F and g have a coupled coincidence point.

Corollary 3.6. *Let $(X, \leq)$ be a partially ordered set and (X, G) be a complete G-metric space. Let $F : X \times X \to X$ and $g : X \to X$ be continuous mappings such that F has the mixed g-monotone property and g commutes with F. Assume that there exist non-negative real numbers a, b with $a + b \in [0,1)$ such that*

$$G(F(x,y), F(u,v), F(w,z)) \leq aG(gx, gu, gw) + bG(gy, gv, gz)$$

for all $x, y, u, v, w, z \in X$ with $gw \leq gu \leq gx$ and $gy \leq gv \leq gz$. Also, suppose that $F(X \times X) \subseteq g(X)$. If there exist $x_0, y_0 \in X$ such that $gx_0 \leq F(x_0, y_0)$ and $F(y_0, x_0) \leq gy_0$, then F and g have a coupled coincidence point.

Proof. We have

$$aG(gx, gu, gw) + bG(gy, gv, gz) \leq (a + b) \max\{G(gx, gu, gw), G(gy, gv, gz)\}$$

for all $x, y, u, v, w, z \in X$ with $gw \leq gu \leq gx$ and $gy \leq gv \leq gz$. Then Corollary 3.6. follows from Corollary 3.5..

Remark 3.1. Note that similar results can be deduced from Theorems 3.2. and 3.3..

Remark 3.2. (1) Theorem 3.1 in [36] is a special case of Theorem 3.1..

(2) Theorem 3.2 in [36] is a special case of Theorem 3.2..

Example 3.1. Let $X = 0,1, 2, 3,...$ and $G : X \times X \times X \to R^+$ be defined as follows:

$$G(x,y,z) = \begin{cases} x+y+z, & \text{if } x,y,z \text{ are all distinct and different from zero,} \\ x+z, & \text{if } x = y \neq z \text{ and all are different from zero,} \\ y+z+1, & \text{if } x = 0, y \neq z \text{ and } y,z \text{ are different from zero,} \\ y+2, & \text{if } x = 0, z = y \neq 0, \\ 1+z, & \text{if } x = 0, y = 0, z \neq 0, \\ 0, & \text{if } x = y = z. \end{cases}$$

Then (X, G) is a complete G-metric space [36]. Let a partial order $\preccurlyeq$ on X be defined as follows: For $x, y \in X$, $x \preccurlyeq y$ holds if $x > y$ and 3 divides $(x - y)$ and $3 \preccurlyeq 1$ and $0 \preccurlyeq 1$ hold. Let $F : X \times X \to X$ *be* defined as follows:

$$F(x,y) = \begin{cases} 1, & \text{if } x \prec y, \\ 0, & \text{if otherwise.} \end{cases}$$

Let $w \preccurlyeq u \preccurlyeq x \preccurlyeq y \preccurlyeq v \preccurlyeq z$ hold, then equivalently, we have $w \geq u \geq x \geq y \geq v \geq z$. Then $F(x, y) = F(u, v) = F(w, z) = 1$. Let $\psi(t) = t, \phi(t) = \left(1 - \frac{k}{2}\right)t$ for $t \geq 0$ and $k \in [0,1)$ and let $g(x) = x$ for $x \in X$. Thus left-hand side of (1) is $G(1, 1,1) = 0$ and hence (1) is satisfied. Then with $x_0 = 81$ and $y_0 = 0$ the Theorem 3.2. is applicable to this example. It may be observed that in this example the coupled fixed point is not unique. Hence, (0,0) and (1,0) are two coupled fixed point of F.

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Authors' contributions
All authors carried out the proof. All authors conceived of the study, and participated in its design and coordination. All authors read and approved the final manuscript.

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